cutoff frequencies between O-guide and circular rod waveguide. A gas-confined dielectric waveguide will have practical application to low-loss transmission lines and high-Q resonators in the submillimeter wavelengths region.

ACKNOWLEDGMENT

The author wishes to thank Dr. Y. Naito of the Tokyo Institute of Technology for invaluable discussions, and Dr. S. Shimada, Dr. H. Kimura, Dr. S. Seki, and Dr. M. Akaike, of the Electrical Communication Laboratories, for their encouragement and guidance.

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Composite Dielectric Waveguides with Two Elliptic-Cylinder Boundaries

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***Abstract*—It is shown that the propagation constants of composite dielectric waveguides with two different elliptic-cylinder boundaries, such as the recent single-polarization optical fibers, are computable by the point-matching method. Numerical results are shown for various combinations of the dielectric constants.**

I. INTRODUCTION

Composite dielectric waveguides, or dielectric-rod structures composed of a few dielectric materials, are expected to be applicable to the transmission of optical waves and microwaves.

Composite dielectric waveguides having two semicircular dielectric regions were discussed in a previous paper [1] where the propagation constants of various transmission modes were computed by applying the point-matching method to interface conditions between two dielectric regions and a microwave model experiment to confirm numerical results was described.

Recent investigations of single-polarization optical fibers [2], [3], have attracted our attention to multicylindrical boundary structures such as an elliptical core with circular cladding and a circular core with elliptical cladding. The detailed analysis of wave propagation characteristics along such dielectric waveguides is complicated compared with one-boundary structures [4]–[6], but it is important.

Ferdinandoff and Bulgarien discussed dielectric waveguides with elliptical cross sections and derived approximate characteristic equations for some extreme cases [7]. However, they have not indicated any numerical data of waveguide characteristics or computational feasibility to support his method.

This paper shows the applicability of the point-matching method to the modal analysis of some composite dielectric waveguides with two elliptic-cylinder boundaries including two crossing boundaries.

II. THE METHOD OF ANALYSIS

The cross-sectional view of various dielectric waveguides is shown in Fig. 1(a)–(j). The first structure, a circular dielectric rod, as shown in Fig. 1(a), is a historical one which was analyzed by Hondros and Debye seventy years ago [4] with the method of the separation of variables. Electromagnetic fields inside and outside of the rod in this case were expressed by using Bessel's and modified Bessel's functions, respectively.

The second structure in Fig. 1(b), a rectangular dielectric rod, can no longer be treated with the method of the separation of variables. Goell [8] applied the point-matching method to the analysis in order to satisfy interface conditions imposed on electromagnetic fields, and used a linear combination of Bessel's functions to express the fields.

This method was employed to analyze the wave propagation along optical fibers with deformed boundaries such as a chipped circle as shown in Fig. 1(c) [5]. The numerical results of this analysis were also confirmed by a microwave model experiment [5].

A composite dielectric waveguide as shown in Fig. 1(d), was analyzed by modifying the point-matching method so as to treat three dielectric regions [1]. A microwave model experiment indicated data supporting the consequent analytical results [1].

Optical fibers of a circular core with an elliptical cladding and those of an elliptical core with a circular cladding as shown in Fig. 1(e) and (f), were fabricated by Kaminow *et al.* [2] and Matsumura *et al.* [3] for maintaining a state of polarization over an extended length. The cross sections of core-cladding structures as shown in Fig. 1(e)–(h) belong to a class of composite dielectric waveguides. These structures can be expressed by a combination of two different elliptic-cylinder boundaries as shown in Figs. 2 and 3.

First, we employ the circular cylindrical coordinate systems (r,θ,z) and assume the propagation factor $\exp(j\omega t-j\beta z)$ in each field function. Then, the fundamental wave equations are given by

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}-\beta^2+k_i^2\right)\begin{Bmatrix}E_{zi}\\H_{zi}\end{Bmatrix}=0,\qquad i=1,2,3 \tag{1}$$

where the suffix 1 denotes the central dielectric region with the highest dielectric constant, the suffix 3 the outside dielectric region, and the suffix 2 the remaining dielectric region. Therefore, the dielectric constants of the three regions are related by

$$\epsilon_1>\epsilon_2 \qquad \epsilon_1>\epsilon_3. \tag{2}$$

Then, the wavenumber of each region is given by

$$k_i^2=\omega^2\epsilon_i\mu_0, \qquad i=1,2,3. \tag{3}$$

The propagation constant β should be in the range

$$k_1>\beta>k_3. \tag{4}$$

Manuscript received November 26, 1980; revised April 1, 1981. This work was supported in part by the 1979 NTT Research Contract on dielectric waveguide analysis.

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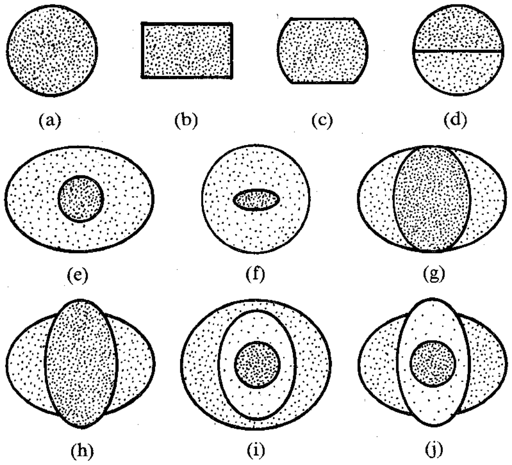

Fig. 1. Cross-sectional view of various dielectric waveguides.

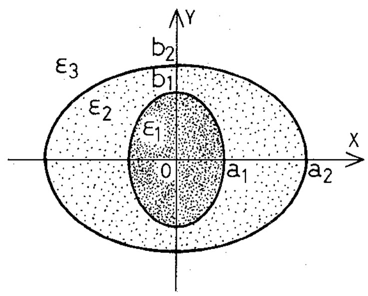

Fig. 2. Cross-sectional view of a composite dielectric waveguide with two elliptic-cylinder boundaries.

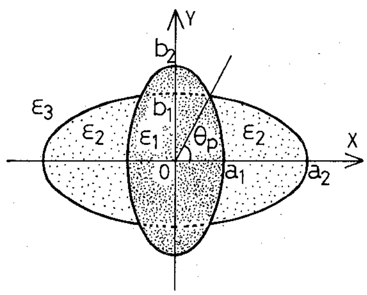

Fig. 3. Cross-sectional view of a composite dielectric waveguide with two crossing elliptic-cylinder boundaries.

Considering this inequality, we define parameters such as

$$h_1^2 = k_1^2 - \beta^2 \tag{5a}$$

$$h_2^2 = \begin{cases} k_2^2 - \beta^2, & \beta < k_2 \\ \beta^2 - k_2^2, & \beta > k_2 \end{cases} \tag{5b}$$

$$h_3^2 = \beta^2 - k_3^2. \tag{5c}$$

General solutions of the wave equations for each region are

$$E_{zi} = \sum_{n=0}^{\infty} [A_{ni}F_{ni}(h_i r) + B_{ni}G_{ni}(h_i r)] \sin(n\theta + \phi_n) \cdot \exp(j\omega t - j\beta z)$$

$$H_{zi} = \sum_{n=0}^{\infty} [C_{ni}F_{ni}(h_i r) + D_{ni}G_{ni}(h_i r)] \sin(n\theta + \psi_n) \cdot \exp(j\omega t - j\beta z), \qquad i = 1,2,3 \tag{6}$$

A_{ni}, B_{ni}, C_{ni}, and D_{ni}, are amplitude coefficients, and ϕ_n and ψ_n are phase angles. These are determined by interface conditions and structural symmetry. $F_{ni}(h_i r)$ and $G_{ni}(h_i r)$ are Bessel's functions, J_n and Y_n, or modified Bessel's functions, I_n and K_n, depending on the size of β. These functions also have singularities either at $r=0$ or $r=\infty$. Therefore, F_{ni} and G_{ni} can be written as

$$F_{n1}(h_1 r) = J_n(h_1 r) \tag{7a}$$

$$F_{n2}(h_2 r) = \begin{cases} J_n(h_2 r), & \beta < k_2 \\ I_n(h_2 r), & \beta > k_2 \end{cases} \tag{7b}$$

$$F_{n3}(h_3 r) = 0 \tag{7c}$$

$$G_{n1}(h_1 r) = 0 \tag{7d}$$

$$G_{n2}(h_2 r) = \begin{cases} Y_n(h_2 r), & \beta < k_2 \\ K_n(h_2 r), & \beta > k_2 \end{cases} \tag{7e}$$

$$G_{n3}(h_3 r) = K_n(h_3 r). \tag{7f}$$

Other components of the electromagnetic fields can be obtained by the following relations derived from Maxwell's equations:

$$E_{ri} = \frac{-j\beta}{k_i^2 - \beta^2}\left[\frac{\partial E_{zi}}{\partial r} + \frac{\omega\mu_0}{\beta r}\frac{\partial H_{hi}}{\partial \theta}\right] \tag{8a}$$

$$E_{\theta i} = \frac{-j\beta}{k_i^2 - \beta^2}\left[\frac{1}{r}\frac{\partial E_{zi}}{\partial \theta} - \frac{\omega\mu_0}{\beta}\frac{\partial H_{zi}}{\partial r}\right] \tag{8b}$$

$$H_{ri} = \frac{-j\beta}{k_i^2 - \beta^2}\left[-\frac{k_i^2}{\omega\mu_0\beta r}\frac{\partial E_{zi}}{\partial \theta} + \frac{\partial H_{zi}}{\partial r}\right] \tag{8c}$$

$$H_{\theta i} = \frac{-j\beta}{k_i^2 - \beta^2}\left[\frac{k_i^2}{\omega\mu_0\beta}\frac{\partial E_{zi}}{\partial r} + \frac{1}{r}\frac{\partial H_{zi}}{\partial \theta}\right]. \tag{8d}$$

Tangential components of electromagnetic-field vectors at any point on dielectric interface can be expressed by combining the r-component and θ-component of the field vectors at that point as

$$E_t = \xi(r,\theta)E_r + \eta(r,\theta)E_\theta \tag{9a}$$

$$H_t = \xi(r,\theta)H_r + \eta(r,\theta)H_\theta \tag{9b}$$

where ξ and η are factors at each point determined by the geometry of dielectric boundaries.

The interface condition to be imposed on the electromagnetic fields is the continuation of the tangential components, but it is not easy to satisfy the condition in this problem without using any approximation. It is felt that the procedure of the point-matching method is particularly suited to this type of boundary structures. Namely, only a finite number of terms N in (6) are chosen and the interface conditions are satisfied only at a finite number of boundary points $2N$. This procedure leads to a set of homogeneous linear equations.

III. Numerical Results

The set of homogeneous linear equations thus obtained has nontrivial solutions only when the determinant of the coefficients vanishes. This determinantal equation is an eigenvalue equation which is numerically solvable on an electronic computer.

It is necessary to treat only the first quadrant of the structure as shown in Fig. 2 because of its symmetry. We have taken the first quadrant of two elliptic-cylinder boundaries given by the

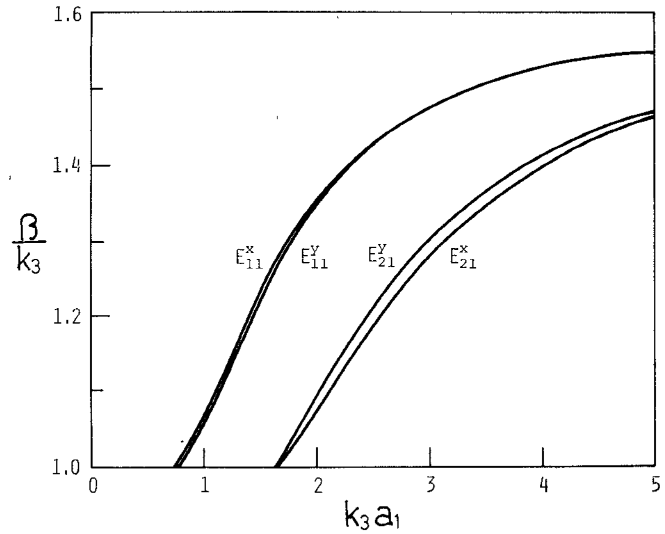

Fig. 4. The computed propagation constant of the composite dielectric waveguide as shown in Fig. 2. $a_1=b_1, a_2=1.5a_1, b_2=1.2a_1, k_1=1.6k_3, k_2=1.3k_3$.

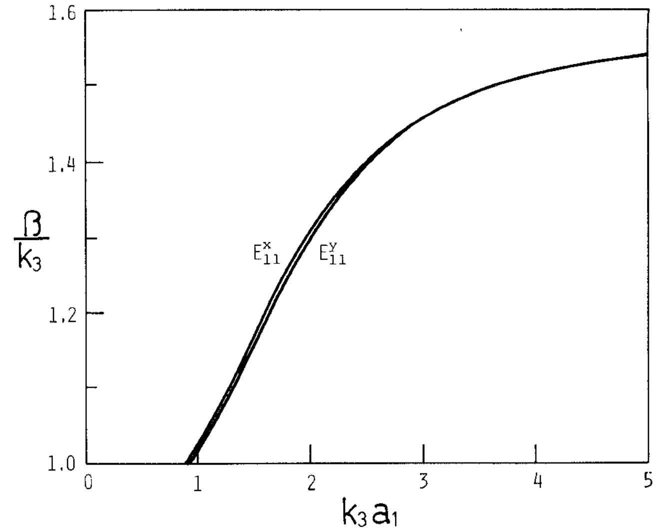

Fig. 6. The computed propagation constant of the composite dielectric waveguide as shown in Fig. 3. $a_1=b_1$, $a_2=1.125a_1$, $b_2=0.9a_1$, $k_1=1.6k_3$, $k_2=1.3k_3$.

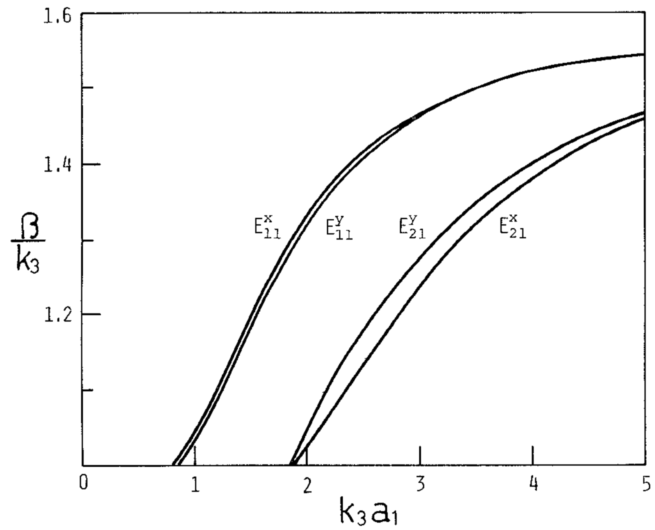

Fig. 5. The computed propagation constant of the composite dielectric waveguide as shown in Fig. 1(g). $a_1=b_1=b_2$, $a_2=1.25a_1$, $k_1=1.6k_3$, $k_2=1.3k_3$.

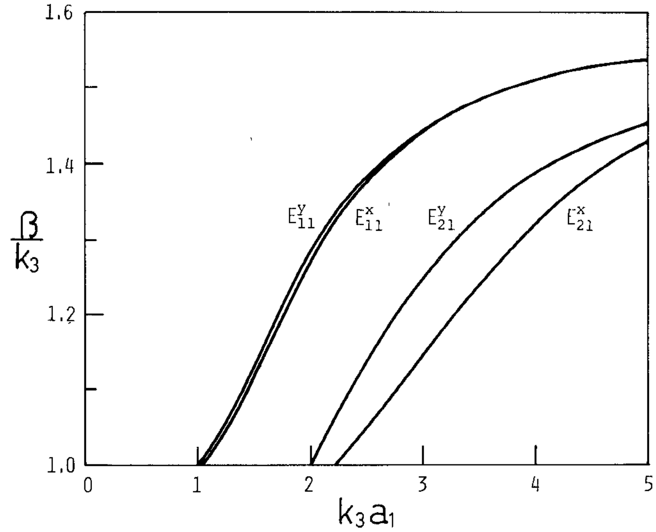

Fig. 7. The computed propagation constant of the composite dielectric waveguide as shown in Fig. 1(g). $a_1=b_1=b_2$, $a_2=1.25a_1$, $k_1=1.6k_3$, $k_2=0.7k_3$.

following expressions:

$$(x/a_1)^2+(y/b_1)^2=1 \tag{10a}$$

$$(x/a_2)^2+(y/b_2)^2=1. \tag{10b}$$

When these boundaries intersect each other, the θ-coordinate of the intersection point is given by

$$\theta_p=\tan^{-1}\left(\frac{b_1b_2}{a_1a_2}\sqrt{\frac{a_2^2-a_1^2}{b_1^2-b_2^2}}\right) \tag{11}$$

as indicated in Fig. 3. This structure has been included in the above analysis.

The first quadrant of the cross section in Fig. 2 or Fig. 3 is divided into equiangle regions to choose matching points on the boundary. The number of matching points is adjusted to obtain a square ($8N\times 8N$) matrix for the coefficients of the linear equations. Numerical results calculated on four example structures are shown in Figs. 4–7. We designate propagation modes as E^x_{mn} or E^y_{mn} according to the Goell's scheme [8].

The numerical convergence of the propagation constant depends on various parameters of waveguide structures. However, we can say that the case of boundary shape closer to the circular, lower order modes, and/or higher frequency leads to faster convergence in general. In Fig. 4, for example, the calculated propagation constants of the dominant E^x_{11} mode are of 5-digit effective numbers for $N=5$ while those of higher order E^x_{21} mode are of 3-digit effective number for $N=12$. The computation time of one value of the propagation constant was about 4 s on the HITAC-M170 computer. Numerical results for a special case, an elliptical dielectric rod waveguide, were also checked with those in a previous paper [5].

Acknowledgment

The authors thank Dr. Y. Suzuki for his helpful comments.

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Patent Abstracts

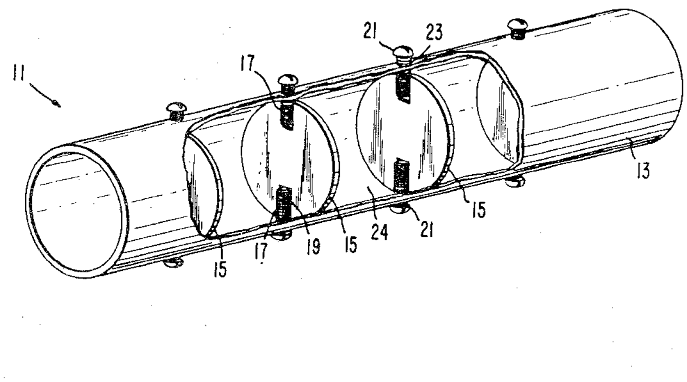
4,251,787 Feb. 17, 1981

Adjustable Coupling Cavity Filter

Inventors: Frederick A. Young; Charles F. Montgomery.
Assignee: Hughes Aircraft Company.
Filed: Mar. 19, 1979.

Abstract—There is herein described a microwave coupled-cavity filter having adjacent cavities defined by spaced end walls disposed in a tubular side wall, the end walls including at least one pair of oppositely disposed coupling apertures extending radially from the side wall toward the center of the associated end wall, and at least one of the associated pair of the coupling apertures containing a tuning screw extending through the side wall for precisely adjusting the coupling between adjacent filter cavities.

7 Claims, 4 Drawing Figures



4,251,786 Feb. 17, 1981

Stepped-Rod Ferrite Microwave Limiter Having Wide Dynamic Range and Optimal Frequency Selectivity

Inventors: Harry Goldie; Steven N. Stitzer.
Assignee: The United States of America as represented by the Secretary of the Air Force.
Filed: Jul. 6, 1979.

Abstract—A coaxial line, wide dynamic range, ferrite limiter having optimal frequency selectivity for microwave frequencies is provided by a stepped ferrite rod with disks of varying volumes and dielectric constants controlling the operating frequency and threshold level for each step segment.

5 Claims, 1 Drawing Figure